Problem 1.5

(a)

$$1 = \int |\Psi|^2 dx = 2|A|^2 \int_0^\infty e^{-2\lambda x} dx = 2|A|^2 \left(\frac{e^{-2\lambda x}}{-2\lambda} \right)\Big|_0^\infty = \frac{|A|^2}{\lambda}; \quad \boxed{A = \sqrt{\lambda}.}$$

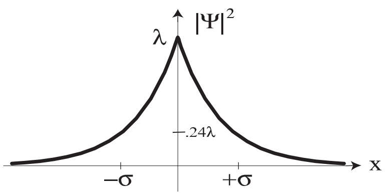
(b)

$$\langle x \rangle = \int x|\Psi|^2 dx = |A|^2 \int_{-\infty}^{\infty} x e^{-2\lambda|x|} dx = \boxed{0.} \qquad \text{[Odd integrand.]}$$

$$\langle x^2 \rangle = 2|A|^2 \int_0^\infty x^2 e^{-2\lambda x} dx = 2\lambda \left[\frac{2}{(2\lambda)^3} \right] = \boxed{\frac{1}{2\lambda^2}.}$$

(c)

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{2\lambda^2}; \qquad \boxed{\sigma = \frac{1}{\sqrt{2}\lambda}.} \qquad |\Psi(\pm\sigma)|^2 = |A|^2 e^{-2\lambda\sigma} = \lambda e^{-2\lambda/\sqrt{2}\lambda} = \lambda e^{-\sqrt{2}} = 0.2431\lambda.$$



Probability outside:

$$2\int_\sigma^\infty |\Psi|^2 dx = 2|A|^2 \int_\sigma^\infty e^{-2\lambda x} dx = 2\lambda \left(\frac{e^{-2\lambda x}}{-2\lambda} \right)\Big|_\sigma^\infty = e^{-2\lambda\sigma} = \boxed{e^{-\sqrt{2}} = 0.2431.}$$

Problem 1.6

For integration by parts, the differentiation has to be with respect to the *integration* variable – in this case the differentiation is with respect to t, but the integration variable is x. It's true that

$$\frac{\partial}{\partial t}(x|\Psi|^2) = \frac{\partial x}{\partial t}|\Psi|^2 + x\frac{\partial}{\partial t}|\Psi|^2 = x\frac{\partial}{\partial t}|\Psi|^2,$$

but this does *not* allow us to perform the integration:

$$\int_a^b x\frac{\partial}{\partial t}|\Psi|^2 dx = \int_a^b \frac{\partial}{\partial t}(x|\Psi|^2) dx \neq (x|\Psi|^2)\Big|_a^b.$$

Quantum Physics 1 - Test 2

Consider a particle of mass m subject to the harmonic oscillator potential ($V(x) = \frac{1}{2}m\omega^2 x^2$), and assume that, at $t = 0$, the particle is in the state

$$\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}.$$

a. Normalize this wave function. (*1 point*)
Solution:
Hint: $\int_{-\infty}^{\infty} e^{-\alpha x^2}\, dx = \sqrt{\frac{\pi}{\alpha}}$.

$$\psi_0(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}.$$

We might as well normalize it right away:

$$1 = |A|^2 \int_{-\infty}^{\infty} e^{-m\omega x^2/\hbar}\, dx = |A|^2 \sqrt{\frac{\pi\hbar}{m\omega}}.$$

so $A^2 = \sqrt{m\omega/\pi\hbar}$**, and hence**

$$\boxed{\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}.}$$

b. Find the first excited state. You do not have to normalize it. (*2 points*)
Hint:The ladder operator is given by $a_+ = \frac{1}{\sqrt{2\hbar m\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)$.
Solution:

$$\psi_1(x) = A_1 a_+ \psi_0 = \frac{A_1}{\sqrt{2\hbar m\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= A_1 \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}}\, x e^{-\frac{m\omega}{2\hbar}x^2}.$$

c. Add time dependence to the initial state, i.e. find $\Psi_0(x,t)$. (*3 points*)
Solution:

$$\Psi(x,t) = Ae^{-\frac{m\omega}{2\hbar}x^2}e^{-i\frac{E_0}{\hbar}t} \tag{1}$$
$$= Ae^{-\frac{m\omega}{2\hbar}x^2}e^{-i\frac{\omega}{2}t}. \tag{2}$$

Here we used that the energy of the ground state of a harmonic oscillator is $E_0 = \hbar\omega/2$.

d. Find the allowed energies of the *half* harmonic oscillator (*3 points*)

$$V_{\frac{1}{2}}(x) = \begin{cases} \frac{1}{2}m\omega^2x^2, & \text{for } x > 0. \\ \infty, & \text{for } x < 0. \end{cases}$$

(This represents, for example, a spring that can be streched, but not compressed.)
Hint: This requires some careful thought, but very little actual computation.
Solution:

Problem 2.42

Everything in Section 2.3.2 still applies, except that there is an additional boundary condition: $\psi(0) = 0$. This eliminates all the *even* solutions ($n = 0, 2, 4, \dots$), leaving only the odd solutions. So

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega,\ n = 1, 3, 5, \dots .}$$

Quantum Physics 1 – Test 3
Solutions

A free particle has the initial wave function

$$\Psi(x,0) = Ae^{-a|x|},$$

where A and a are positive real constants.

a. *Normalize $\Psi(x,0)$. (1 point)*

$$1 = \int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx = 2|A|^2 \int_0^{\infty} e^{-2ax} dx = 2|A|^2 \frac{e^{-2ax}}{-2a}\Big|_0^{\infty} = \frac{|A|^2}{a} \Rightarrow A = \boxed{\sqrt{a}}$$

b. *Sketch a graph of $|\Psi(x,0)|^2$. (1 point)*

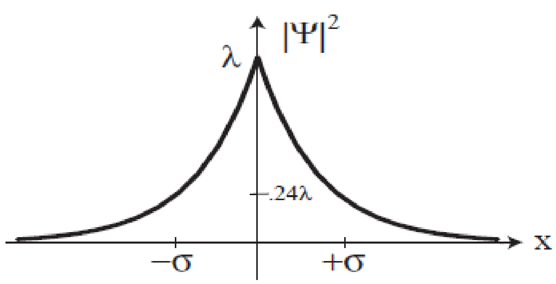


Figure 1: Graph of $|\Psi(x,0)|^2$. Imagine λ is a.

c. *Find $\phi(k)$. (4 points)*
Hint: $\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0)e^{-ikx}\mathrm{d}x$

$$\begin{aligned}
\phi(k) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \sqrt{a}e^{-|a|x}e^{-ikx}\mathrm{d}x \\
&= \sqrt{\frac{a}{2\pi}} \left[\int_{-\infty}^{0} e^{ax-ikx}\mathrm{d}x + \int_0^{+\infty} e^{-ax-ikx}\mathrm{d}x\right] \\
&= \sqrt{\frac{a}{2\pi}} \left[\frac{1}{a-ik}e^{x(a-ik)}\Big|_{-\infty}^{0} + \frac{1}{-a-ik}e^{-x(a+ik)}\Big|_0^{\infty}\right] \\
&= \sqrt{\frac{a}{2\pi}} \left[\frac{1}{a-ik} + \frac{1}{a+ik}\right] = \sqrt{\frac{a}{2\pi}} \left[\frac{a+ik}{a^2+k^2} + \frac{a-ik}{a^2+k^2}\right] \\
&= \boxed{\sqrt{\frac{a}{2\pi}}\frac{2a}{a^2+k^2}}
\end{aligned}$$

d. *Discuss the limiting cases where a is very small or very large. What can you say about the position and momentum in these cases? are they well- or ill-defined? (*3 points*)*

For *large* a, $\Psi(x, 0)$ is a sharp narrow spike whereas $\phi(k) \simeq \sqrt{2/a\pi}$ is broad and flat; position is well-defined and momentum is ill-defined. For *small* a, $\Psi(x, 0)$ is broad and flat, whereas $\psi(k) \simeq (\sqrt{2a^3/\pi})/k^2$ is a sharp and narrow spike; position is ill-defined and momentum is well-defined.

Quantum Physics 1 - Test 1

Consider the potential

$$V(x) = \begin{cases} 0, & \text{for } x < a \\ V_0, & \text{for } -a \leq x \leq a \\ 0, & \text{for } x > a \end{cases}$$

Where V_0 is a positive real constant.

a. Consider a particle on the left side of the barrier (so $x < -a$), traveling in the $+x$-direction. The energy of the particle is smaller than V_0. Is it possible to find the particle on the right side of the barrier according to classical physics? According to quantum physics? If there is a difference, what phenomenon causes this? *(1 points)*
Solution: For classical no, for quantum yes. Due to quantum tunneling

b. Write down the three-piece wave function that matches the potential $V(x)$ for the scattering states. *(3 points)*
Solution: Need to find different exponents and see that it vanishes in certain domains.

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < -a \\ C\sin lx + D\cos lx & -a < x < a \\ Fe^{ikx} + Ge^{-ikx} & x > a \end{cases}$$

$$l = \frac{\sqrt{2m(E - V_0)}}{\hbar}, \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

1 point for the domains where V=0,
1 point for the domain where V=V0,
1 point for the correct k,l expressions.
It is also correct if the assume no incoming waves from the right (so if they set G=0).

c. Using (dis)continuity of the wave function at the boundaries, we find that

$$T^{-1} = 1 + \left[\frac{(k^2 + l^2)^2}{(2kl)^2} - 1\right]\sin^2(2la)$$

For what energies will perfect transmission occur? *(1 points)*
Solution: Ei = V0 + E(infinite square well). Can also be seen from T by setting sin=0 (1 point for the final solution)

Consider the operator

$$\hat{Q} \equiv i\frac{d}{d\phi}$$

where ϕ is the usual polar coordinate in two dimensions.

a. Is $\hat{Q}$ hermitian? *(2 points)* **1 point for knowing condition hermitian**
1 point for correct calculation.

b. Find its eigenfunctions and eigenvalues. *(2 points)* **1 point for eigenfunction**
1 for eigenvalues.

Solution: See example 3.1 book.

Quantum Physics 1 - Test 5 solutions

a. Calculate $[p^2, x]$ (*1 points*)
Solution:

$$\left[p^2, x\right] = p[p, x] + [p, x]p = -2i\hbar p$$

b. Prove the virial theorem,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle xp\rangle = 2\langle T\rangle - \left\langle x\frac{\mathrm{d}V}{\mathrm{d}x}\right\rangle$$

where $T = \frac{\hat{p}^2}{2m}$ is the kinetic energy ($H = T + V$). (*3 points*)
Hint: use that

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle Q\rangle = \frac{i}{\hbar}\left\langle\left[\hat{H}, \hat{Q}\right]\right\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$$

Solution:
Filling in the equation in the hint gives:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\langle xp\rangle &= \frac{i}{\hbar}\langle[H, xp]\rangle + \frac{\partial xp}{\partial t} \\
[H, xp] &= \left[\frac{p^2}{2m}, xp\right] + [V(x), xp] \\
&= \frac{1}{2m}\left[p^2, x\right]p + x[V(x), p] \\
&= -\frac{1}{2m}2i\hbar p^2 + i\hbar x\frac{\mathrm{d}V(x)}{\mathrm{d}x} \\
\implies \frac{\mathrm{d}}{\mathrm{d}t}\langle xp\rangle &= \frac{i}{\hbar}\left\langle -2i\hbar\frac{p^2}{2m} + i\hbar x\frac{\mathrm{d}V}{\mathrm{d}x}\right\rangle \\
&= 2\langle T\rangle - \left\langle x\frac{\mathrm{d}V}{\mathrm{d}x}\right\rangle
\end{aligned}$$

c. What does this reduce to for stationary states? (*1 point*)
Solution: Expectation values of stationary states time-independent, so $\frac{\mathrm{d}}{\mathrm{d}t}\langle xp\rangle = 0$, so $2\langle T\rangle = \left\langle x\frac{\mathrm{d}V}{\mathrm{d}x}\right\rangle$

d. For the harmonic oscillator ($V = \frac{1}{2}m\omega^2x^2$), find the relation between $\langle T\rangle$ and $\langle V\rangle$ (*1 point*)
Solution:
Then $\left\langle x\frac{\mathrm{d}V}{\mathrm{d}x}\right\rangle = \langle x \cdot m\omega^2 x\rangle = 2\langle V\rangle$, so for stationary states $\langle T\rangle = \langle V\rangle$. (Non-stationary states would have $\langle T\rangle = \langle V\rangle + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\langle xp\rangle$.)

e. Are the following statements true or false? (*3 points*)

1. A state cannot be a simultaneous eigenstate of two compatible observables. *False*
2. Hermitian operators have real eigenvalues in finite-dimensional vector spaces. *True*
3. All wavefunctions are linear combinations of eigenfunctions of a Hermitian operator. *True*
4. The generalized uncertainty principle is one of the central assumptions of quantum mechanics. *False*
5. If a matrix is equal to the complex conjugate of its transpose, then the matrix is Hermitian. *True*
6. Not all observables are Hermitian. *False*

Quantum Physics 1 - Test 6
Solution

Consider the three-dimensional harmonic oscillator, for which the potential is

$$V(r) = \frac{1}{2} m\omega^2 r^2. \tag{1}$$

a. Show that the separation of variables in cartesian coordinates turns this into three one-dimensional oscillators. (*2 points*)

Hint: Time-independent Schrödinger equation for the one-dimensional harmonic oscillator

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{d^2x} + V\psi = E\psi \tag{2}$$

b. Derive the allowed energies of the total wavefunction of the three-dimensional harmonic oscillator using what found in the previous point. (*2 points*)

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \frac{1}{2}m\omega^2\left(x^2+y^2+z^2\right)\psi = E\psi.$$

Let $\psi(x,y,z) = X(x)Y(y)Z(z)$; plug it in, divide by XYZ, and collect terms:

$$\left(-\frac{\hbar^2}{2m}\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{2}m\omega^2x^2\right) + \left(-\frac{\hbar^2}{2m}\frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{2}m\omega^2y^2\right) + \left(-\frac{\hbar^2}{2m}\frac{1}{Z}\frac{d^2Z}{dz^2} + \frac{1}{2}m\omega^2z^2\right) = E.$$

The first term is a function only of x, the second only of y, and the third only of z. So each is a constant (call the constants E_x, E_y, E_z, with $E_x + E_y + E_z = E$). Thus:

$$-\frac{\hbar^2}{2m}\frac{d^2X}{dx^2} + \frac{1}{2}m\omega^2x^2X = E_xX;\ -\frac{\hbar^2}{2m}\frac{d^2Y}{dy^2} + \frac{1}{2}m\omega^2y^2Y = E_yY;\ -\frac{\hbar^2}{2m}\frac{d^2Z}{dz^2} + \frac{1}{2}m\omega^2z^2Z = E_zZ.$$

Each of these is simply the one-dimensional harmonic oscillator (Eq. 2.44). We know the allowed energies (Eq. 2.61):

$$E_x = (n_x + \tfrac{1}{2})\hbar\omega;\ E_y = (n_y + \tfrac{1}{2})\hbar\omega;\ E_z = (n_z + \tfrac{1}{2})\hbar\omega;\ \text{where } n_x, n_y, n_z = 0, 1, 2, 3, \ldots.$$

So $E = (n_x + n_y + n_y + \frac{3}{2})\hbar\omega = \boxed{(n + \frac{3}{2})\hbar\omega,}$ with $n \equiv n_x + n_y + n_z$.

c. Given the ground state of the three-dimensional harmonic oscillator

$$\psi_0(r) = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4} e^{-\frac{m\omega}{2\hbar}r^2}, \tag{3}$$

compute $\langle r\rangle$. (*2 points*)

Hint:

$$\int_0^\infty \mathrm{d}x\, x^{2n+1} e^{-x^2/a^2} = \frac{n!}{2}a^{2n+2}. \tag{4}$$

$$
\begin{aligned}
\langle r \rangle &= \int \mathrm{d}V \, r |\psi_0(r)|^2 \\
&= \left(\frac{m\omega}{\pi\hbar}\right)^{3/2} \int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{\infty} \mathrm{d}r r^3 e^{-\frac{m\omega}{\hbar} r^2}
\end{aligned} \tag{5}
$$

and using the hint for $n = 1$ and $a = \sqrt{\hbar/m\omega}$

$$
\langle r \rangle = 4\pi \left(\frac{m\omega}{\pi\hbar}\right)^{3/2} \frac{1}{2} \left(\frac{\hbar}{m\omega}\right)^2 = 2\sqrt{\frac{\hbar}{\pi m\omega}}. \tag{6}
$$

d. Are the following statements true or false? (*3 points*)

1. The ground state of the system above has non-zero $\langle \mathbf{p} \rangle$. *False, according to the Ehrenfest's theorem.*
2. The first excited state of the system above is 3-fold degenerate. *True, first excited state is given by* $(n_x, n_y, n_z) = (1\,0\,0)$, $(0\,1\,0)$, $(0\,0\,1)$.
3. The Coulomb potential that describes proton-electron interaction admits both continuum and bound states. *True, continuum for scattering and bound for the hydrogen atom.*

Quantum Physics 1 - Test 7

An electron is in the spin state

$$\chi = A\begin{pmatrix}1-2i\\2\end{pmatrix}.$$

a. Determine the constant A by normalizing χ. (*1 point*)
Solution:

$$1 = \chi^\dagger\chi = A^*A\begin{pmatrix}1+2i & 2\end{pmatrix}\begin{pmatrix}1-2i\\2\end{pmatrix} = |A|^2(1+4+4) = 9|A|^2$$
$$\implies |A| = \frac{1}{3}.$$

b. If you measured S_z on this electron, what values could you get, and what is the probability of each value? What is the expectation value of S_z? (*3 points*)
solution:
The possible values for an electron are $\pm\frac{\hbar}{2}$, and the possibility for each is given by the coefficient of its eigenvector. Decomposing χ into the up and down eigenvectors we find

$$\chi = \frac{1}{3}\left(\begin{pmatrix}1-2i\\2\end{pmatrix}\right) = \frac{1}{3}\left((1-2i)\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix}\right) = c_+\chi_+ + c_-\chi_-,$$

such that $p_+ = c_+^*c_+ = \frac{5}{9}$ and $p_- = \frac{4}{9}$ are the probabilities to find the electron in the state with $s_z = +\frac{\hbar}{2}$ and $s_z = -\frac{\hbar}{2}$ respectively. The expectation value is given by $\langle S_z\rangle = \frac{5}{9}\left(\frac{\hbar}{2}\right) + \frac{4}{9}\left(-\frac{\hbar}{2}\right) = \frac{\hbar}{18}$.

c. Show that the normalized eigenspinors of S_x are given by $\chi_+^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$.
(*2 points*)
Hint: $S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$.
solution:

Not surprisingly, the possible values for S_x are the same as those for S_z. The eigenspinors are obtained in the usual way:

$$\frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \pm\frac{\hbar}{2}\begin{pmatrix}\alpha\\\beta\end{pmatrix} \Rightarrow \begin{pmatrix}\beta\\\alpha\end{pmatrix} = \pm\begin{pmatrix}\alpha\\\beta\end{pmatrix},$$

so $\beta = \pm\alpha$. Evidently the (normalized) eigenspinors of $\mathbf{S}_x$ are

$$\chi_+^{(x)} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}, \left(\text{eigenvalue} + \frac{\hbar}{2}\right); \quad \chi_-^{(x)} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}, \left(\text{eigenvalue} - \frac{\hbar}{2}\right). \text{ [4.151]}$$

d. Suppose we reset the system to its original state, if you measured S_x on the electron, what values could you get, and what is the probability of each value? What is the expectation value of S_x? *(3 points)*

solution:

The probability of finding $s_x = \pm\frac{\hbar}{2}$, is given by the square of the size of the coefficients of the eigenvectors $\chi_\pm^{(x)}$ when decomposing χ in that basis. This is done by finding the 'overlap' of the eigenvectors with χ:

$$c_+^{(x)} = \left(\chi_+^{(x)}\right)^\dagger \chi = \frac{1}{3}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1-2i\\ 2\end{pmatrix} = \frac{1}{3\sqrt{2}}(1-2i+2) = \frac{3-2i}{3\sqrt{2}}; \quad |c_+^{(x)}|^2 = \frac{9+4}{9\cdot 2} = \frac{13}{18}.$$

$$c_-^{(x)} = \left(\chi_-^{(x)}\right)^\dagger \chi = \frac{1}{3}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}1-2i\\ 2\end{pmatrix} = \frac{1}{3\sqrt{2}}(1-2i-2) = -\frac{1+2i}{3\sqrt{2}}; \quad |c_-^{(x)}|^2 = \frac{1+4}{9\cdot 2} = \frac{5}{18}.$$

$$\boxed{\frac{\hbar}{2}, \text{ with probability } \frac{13}{18}; \ -\frac{\hbar}{2}, \text{ with probability } \frac{5}{18}.} \quad \langle S_x\rangle = \frac{13}{18}\frac{\hbar}{2} + \frac{5}{18}\left(-\frac{\hbar}{2}\right) = \boxed{\frac{2\hbar}{9}.}$$

Quantum Physics 1 - Test 8

1. Consider two electrons (spin 1/2).

a. Give the two-electron spin states ($|s\,m\rangle$) in terms of the one-electron spin states. i.e. give the triplet and singlet states. *(2 points)*
Solution:

$$s = 1 \text{ (triplet)} \;=\; \begin{cases} |1\,1\rangle & = \uparrow\uparrow \\ |1\,0\rangle & = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow + \downarrow\uparrow\right) \\ |1\,-1\rangle & = \downarrow\downarrow \end{cases}$$

$$s = 0 \text{ (singlet)} \;=\; \begin{cases} |0\,0\rangle & = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow - \downarrow\uparrow\right) \end{cases}$$

b. What are entangled state in general *and* which of the states of a. are entangled? *(2 points)*
Solution: in the $|1\,0\rangle$ and $|0\,0\rangle$ states.

c. Consider the two hydrogen atoms which form a covalent bond by sharing their electrons. Do all spin states found in a. allow for this configuration? Motivate you answer *(2 points)*
Solution: The complete state includes the position wavefunction and the spinor (see Eq. 5.23). Under the symmetrization requirement, the spatial part is antisymmetric. The singlet spin state is antisymmetric, the triplet states are symmetric. For bonding we need a symmetric total wavefunction, thus only the singlet spin state is allowed.

2. For the following statements write down "true" when the statement is correct and "false" when the statement is incorrect. Motivate your answer with a single sentence. *(3 points)*

- Two electrons cannot occupy the same state.
  True: Fermions, Pauli exclusion principle
- Nickel ($^3F_4$) has the quantum numbers: $S = 3$, $L = 3$, $J = 4$
  Solution: False, $2S + 1 = 3$ so $S = 1$. Also $L + S \neq J$
- Quantum physics contains local hidden variables
  Solution: False, see Bell's theorem